DOCUMENT RESUME

ED 450 149 TM 032 339

AUTHOR Guthrie, Abbie
TITLE Using Bootstrap Methods with Popular Statistical Programs.
PUB DATE 2001-02-00
NOTE 25p.; Paper presented at the Annual Meeting of the Southwest Educational Research Association (New Orleans, LA, February 1-3, 2001).
PUB TYPE Reports - Descriptive (141) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Computer Software; Heuristics; *Statistical Analysis; Statistical Distributions
IDENTIFIERS AMOS Computer Program; *Bootstrap Methods; Statistical Package for the Social Sciences

ABSTRACT

Developed by B. Efron (1979) and his colleagues (P. Diaconis and B. Efron, 1983), bootstrap methods have the goal of creating an empirical sampling distribution that can be used to test statistical hypotheses, estimate standard errors, and create confidence intervals. Bootstrapping methods offer a unique and effective method for testing the stability and replicability of results. This paper explains the bootstrap method of exploring replicability internally, including a heuristic example applying bootstrap methods to a confirmatory factor analysis, using the Statistical Package for the Social Sciences and AMOS. (Contains 1 figure, 5 tables, and 13 references.) (Author/SLD)

Reproductions supplied by EDRS are the best that can be made
from the original document.

ED 450 149

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY
A. Guthrie
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Using Bootstrap Methods with Popular Statistical Programs

Abbie Guthrie

Texas A&M University 77843-4225

TM032339

Paper presented at the annual meeting of the Southwest Educational Research Association, New Orleans, February 1-3, 2001.

BEST COPY AVAILABLE

Abstract

Social scientists are confronted with the task of researching idiosyncratic entities and constructs that exist in a variable universe. As a result, explaining the results of social science research is a daunting task. Nevertheless, all scientists hope to find answers to research questions that are true, and therefore replicable. Unfortunately, evidence of replicability is often missing from reported research. Developed by Efron (1979) and his colleagues (Diaconis & Efron, 1983), bootstrap methods have the goal of creating an empirical sampling distribution that can be used to test statistical hypotheses, estimate standard errors, and create confidence intervals. Bootstrapping methods offer a unique and effective method for testing the stability and replicability of results. This paper explains the bootstrap method of exploring replicability internally, including a heuristic example applying bootstrap methods to a confirmatory factor analysis, using SPSS and AMOS.

Using Bootstrap Methods with Popular Statistical Packages

Social scientists are confronted with the task of researching idiosyncratic entities and constructs that exist in a variable universe. As a result, explaining the results of social science research is a daunting task. Often, there are several plausible explanations for the results that have been obtained. Any single study is limited in its capacity to explain the complex phenomena that are usually the subject of social science research. Nevertheless, all scientists hope to find answers to research questions that are true, and therefore replicable. Thompson (1995) noted, "the most promising strategies emphasize interpretation based on the estimated likelihood that results will replicate" (p. 86).

Unfortunately, evidence of replicability is often missing from reported research. This is in no small part due to researchers' misconceptions that statistical significance tests to provide evidence of both importance and replicability. In fact, statistical significance tests do neither (Cohen, 1994; Thompson, 1994b, 1995). This topic has received much attention in recent methodology literature and remains controversial. The APA Task Force on Statistical Inference (Wilkinson & APA Task Force, 1999) recommended requiring researchers to address the stability of their results by methods such as effect size reporting (e.g., R^2, eta^2) and comparing results to previous studies.

The only true way to know the replicability of a study is to draw a new sample and do the study again. This is called "external" replication and is rarely, if ever, done in the social sciences. Other

methods of exploring replicability "internally" are available. Rodgers (1999) provides a very accessible overview of the bootstrap, the jackknife, and the randomization test. The present paper will explain the bootstrap method of exploring replicability internally, including a heuristic example applying bootstrap methods to a confirmatory factor analysis, using SPSS and AMOS.

Understanding the Sampling Distribution

Before explaining the particulars of the bootstrap method, a brief review of the different types of distributions used in research is prudent. There are three types of distributions that one needs to understand in order to understand the role of the bootstrap: the population distribution, the sample distribution, and the sampling distribution.

Picture a histogram of scores, with one asterisk for each person in the population of interest. This is the population distribution. Calculations that characterize the population (e.g., means, variances, ranges, correlations) are called parameters. However, the entire population of interest is rarely available for study, because most researchers want to generalize their results to the widest possible population. Therefore, the researcher draws a sample of the population.

The histogram of scores for this sub-group of the population is the sample distribution. Statistics are calculations that characterize a sample. No two samples will be exactly the same. The uniqueness that characterizes a sample is called sampling error. Statistical significance tests evaluate the probability that the difference

between statistics is due to sampling error. [An important point to remember is that, as Cohen (1994) pointed out, the population used in statistical significance testing is presumed to be one in which the null hypothesis is perfectly true (e.g., no difference between groups, no treatment effect).] The estimation of this probability invokes the sampling distribution.

Although both the population and the sample consist of scores, the sampling distribution consists instead of sample statistics estimating population parameters. [A hybrid special case arises when (a) the statistic of interest is the mean and (b) n=1, because in this case each sample statistic in the sampling distribution is also a statistic from the population.] So, we must specify what parameter we are estimating in order to carry forward this discussion. Let's presume that the parameter of interest is the score variance.

Imagine that you could take every possible sample of size n from the population, administer a measurement, and calculate the variance. Each time you calculate the variance for a sample you plot that variance on a histogram. When you finish calculating and plotting the variance for each and every possible combination of n people (or whatever you are measuring), you have a sampling distribution--a histogram of statistics (in this case, variances) obtained from repeated samplings, that models the sampling error.

Each entry for a parameter estimate that is used to create the sampling distribution is based on exactly the same sample size, n, that was the sample size of the actual sample. We must use exactly this sample size for every estimate employed in creating the sampling

distribution, because we are modeling how the sampling error affects the parameter estimates, and the amount of sampling error is largely influenced by the sample size, with more sampling error occurring as n is smaller.

Each sampling distribution is unique to both sample size and the particular statistic being estimated. For example, the sampling distribution of variances for samples of 25 is different than one of variances for samples of 26, as well as being different from a sampling distribution of means for samples of 25.

There are three types of sampling distributions (Rodgers, 1999). The one described above is called the "idealized" sampling distribution, and is made up of all possible resamples from the actual population. However, researchers usually do not have access to the entire population. If they did, they would not be using a sample. Therefore, science has attempted to model the idealized sampling distribution two ways, theoretically and empirically.

Social scientists are no doubt familiar with statistical significance tests such as F and t. These tests invoke "theoretical" sampling distributions. These distributions are mathematically derived and are said to represent the idealized distribution if certain assumptions are met (e.g., normal distribution, equality of variance). An analogy to using a theoretical sampling distribution is buying clothes "off the rack". These clothes come in many sizes and styles, but they are constructed with the assumption that people of a certain size will all be symmetrical and have the same basic measurements of height and width.

In contrast, the second way of modeling the idealized distribution is to create an "empirical" sampling distribution. This is analogous to "haute couture", where clothes are made to your exact measurements and specifications. Reordering or redrawing the original sample derives an empirical sampling distribution from the data in hand (Rodgers, 1999). There are several advantages to using an empirical sampling distribution. For instance, the only assumption required for the use of an empirical sampling distribution is that the researcher believes the sample to be representative of the population, making this technique especially useful for sample data that does not conform to normality assumptions. Also, theoretical sampling distributions are not available for many statistics of interest (e.g., modern statistics). An empirical sampling distribution can be created for any statistic of interest to the researcher (Thompson, 1994a).

Sir Ronald Fisher, who developed the F distribution, was convinced of the superiority of the empirical sampling distribution to the theoretical sampling distribution. Rodgers (1999) reported some of the relevant history and states, "Fisher felt that a test using a theoretical sampling distribution was valid only to the extent that it matched the results that would be obtained using an empirical sampling distribution" (p. 442).

Rodgers suggested that researchers' reliance on test statistics (and therefore theoretical sampling distributions) is only an accident of time. Had computers been available to assist in creating the empirical sampling distribution, the need for theoretical distributions would have disappeared. Just as the haute couture method

of designing clothing is incredibly expensive, empirical sampling distributions are computationally expensive. However, with modern computers, an empirical sampling distribution can be created within seconds at essentially no cost.

Understanding the Bootstrap Method

Developed by Efron (1979) and his colleagues (Diaconis & Efron, 1983), bootstrap methods have the goal of creating an empirical sampling distribution that can be used to test statistical hypotheses, estimate standard errors, and create confidence intervals. Diaconis and Efron explained that the name "bootstrap" is derived from the old saying about pulling yourself up by your own bootstraps, reflecting "the fact that the one available sample gives rise to many others" (p. 120). Thompson (1995) explained the method:

> Conceptually, these methods involve copying the data set many times into an infinitely large "mega" data set. Then hundreds or thousands of different samples are drawn from the "mega" file and results are computed separately for each sample and then averaged. (p. 86)

What actually happens is sampling with replacement. This method creates an environment where a person from the original sample could be drawn more than once in a given resample or not at all, but all resamples will have the same number as the original sample. The resulting empirical sampling distribution "informs the researcher regarding the extent to which results generalize across different types of samples" (Thompson, 1995, p. 86).

For our heuristic example, this paper used cognitive data on 301 students (Holzinger & Swineford, 1939, pp. 81-91) to illustrate a bootstrap of a confirmatory factor analysis (CFA). The model for the analysis is presented in Figure 1. The analyses were run using SPSS 9.0 and Amos 4.0. Arbuckle (1997) recommends fixing the regression paths in the model, as opposed to fixing the variances. He warns that fixing the variances when performing bootstrap replications could result in inflated estimates of standard errors.

Four separate analyses were run. First, a CFA of the model was performed on the entire population of 301 students, using 200 bootstrap resamples. For purposes of comparison, we will treat the 301 students as our population of interest, the one to which we want to generalize. This data will serve as our population distribution. Then, three separate CFA's were performed on a random sample of 75 students with 10, 200, and 2000 bootstrap resamples respectively.

As mentioned previously, in the bootstrap resampling process, an individual may be selected more than once, or not at all, in any given resample. Using Amos, you can request a summary of the bootstrap samples. The output of this request resembles the data presented in the Appendix. For each bootstrap sample, a list of integers is displayed. The list should be read from left to right, beginning with the first row. The first integer tells how often the first person in the original sample appeared in the bootstrap sample. The second integer tells how often the second person appeared, and so on.

The careful reader will notice that "Bootstrap Sample 2" and "Bootstrap Sample 6" appear twice, with different numbers in each set.

This is because Amos was unable to find a solution to two of the samples, thus drawing two more, for a total of twelve resamples drawn to obtain 10 usable samples. This information can be used to compute the average number of times people were used in the resamples, which should very nearly equal the number of resamples if the process was truly random.

Using the bootstrap methods with multivariate techniques, such as CFA, requires some special considerations. Thompson (1994a, 1995) provides a thorough explanation of this topic. Because factors tend to fluctuate in their orders over samples, Factor I might reflect Speed in one sample and Memory in the next. As Thompson (1995) wryly noted:

> If the analyst computed the mean structure (or pattern) coefficients for the first variable on the first component across all the repeated samplings, the mean would be a nonsensical mess representing an average of some apples, some oranges, and perhaps some kiwi. (pp. 88-89)

To provide control, Procrustean methods are used to rotate all factors into a common factor space. Declaring a target matrix can do this. A target matrix might be made up of (a) ones, zeroes, and negatives one's modeling a simple structure based on theoretical expectations, (b) a structure or weight matrix from previous research or the data in hand, or (c) a graphic model such as ours, similar to those used in structural equation modeling.

Exploring Bootstrap Results in Amos

In the text output, following the Maximum Likelihood Estimates, Amos prints out the Bootstrap Results. First, is a Summary of Bootstrap Iterations, followed by the Summary of Bootstrap Samples (as in the Appendix), if this has been requested. Then, Amos provides the Bootstrap Standard Errors. This section presents a summary of the statistics from the empirical sampling distributions that were created by the bootstrapping process. Table 1 summarizes some of these results for the population. For example, the beta (B) weight (pattern coefficient) for the Speeded Addition Test (spdadd) reported is the mean value from the empirical sampling distribution of 200 pattern coefficients over 200 bootstrap samples. Table 1 reports statistics from 12 different empirical sampling distributions (one for each variable, for each statistic). All in all, 28 separate empirical sampling distributions were created by Amos to estimate the reported bootstrap results, each consisting of 200 estimates.

The standard error (SE) reported is the standard deviation of the empirical sampling distribution. The SE is an extremely important statistic in all inferential analyses (Thompson, 1994a). The ratio of a statistic to the SE of a theoretical sampling distribution is variously known as t, F, critical ratio, and Wald's statistic. Bootstrap estimates of SE can also be used for inferential purposes, but the have an added benefit that theoretical SE's do not offer. Bootstrap SE estimates give the researcher an idea of the stability of the estimates over hundreds (or thousands) of configurations of the sample population. In spite of commonly held beliefs and practices, it

is this descriptive use of the SE that speaks to replicability, not the inferential use described above.

If the sample statistic is relatively equal to the mean bootstap estimate, and the SE is small in relation to the mean bootstrap estimate, then the sample statistic can be thought of as stable and, therefore, more likely to replicate. On the other hand, if the SE is large in relation to the mean bootstrap estimate or the difference between the mean bootstrap estimate and the sample statistic reflects a great deal of bias, then caution interpreting the sample data is warranted, as the results show instability. To further assist the researcher with this task, Amos provides Bias estimates and the SE of the SE along with the other bootstrap estimates.

Because maximum likelihood estimates (the default in Amos) assume normality in the distribution of the sample, a noticeable difference in the original estimate and the bootstrap estimates might indicate a deviation of the sample distribution from normal. Table 2 gives an overview of the descriptive statistics for the population and the sample. A large SE could indicate the presence of problems in the distribution due to sampling error. For instance, Tables 3, 4, and 5 present the original and bootstrapped estimates for the pattern coefficients, the squared multiple correlations, and the error variances for comparison. Even the most cursory perusal will draw the eye to the measure variable "memnumb". Bootstrap estimates of all of the statistics for "memnumb" are anomalous in relation to the other variables. The results themselves seem impossible, including negative error variances and R^2 estimates of over 400. Explaining the source of

these anomalies is beyond the scope of this paper (and this writer). However, clearly there is a problem with this variable that needs to be examined. More important, the problem is not apparent in the original estimates, only in the bootstrapped estimates.

Because, in this heuristic example, we have the luxury of knowing our population parameters, we can see how well our sample did at estimating the characteristics present in the population. A comparison of Table 1 and Table 2 illustrates that while the sample did a moderately good job at representing the parameters for the Speed factor, the results are quite different for the Memory factor. However, pattern coefficients are often notoriously unstable across changes in the sample (Thompson, 1994b). Yet, comparing Table 1 with Table 3, we see that "spdadd", "spdcount", "memnumb", and "memshape" all vary noticeably from the population parameters. Using the sample estimates alone, a researcher might easily have drawn erroneous conclusions about the population.

In fact, we do not need to compare to Table 1. By investigating the R^2 estimates and the SE's for these variables, we notice that the SE estimates are nearly equal to or greater than the R^2 estimates on those variables. This is indicative of instability. Bootstrapping methods are the only way to get an estimate of standard error for multiple correlations, as Amos does not provide them otherwise.

Another use of the empirical sampling distribution that speaks both to replicability and importance is the creation of confidence intervals. The earlier mentioned APA Task Force on Statistical Inference (Wilkinson & APA Task Force, 1999), also strongly

recommended reporting of confidence intervals to address the stability of the results obtained and to give the reader a context for interpretation of results. These suggestions will likely become requirements for all APA journals in the near future. Several (i.e., 13) journals are already requiring the reporting of effect sizes (Vacha-Haase, Kogan, & Thompson, 2000). If requested, Amos will provide confidence intervals for any statistic that has a bootstrap estimate.

Advantages and Limitations

Bootstrapping is a unique and effective method for testing the stability and replicability of results. While the bootstrap results of our sample data caused concern about the stability of our results, the same method could strengthen readers' (and editors') confidence in results that are found to be stable. Remember, the original Holzinger and Swineford (1939) study used the entire sample of 301 students. The bootstrapped SE estimates on that study (after 200 resamples) range from .05-.09!

The advantages of using bootstrap methods include not having to conform to distributional assumptions, or worry that violating assumptions somehow affected the outcome of the analysis. This paper presented one example applying bootstrap methods to a CFA with one model. Bootstrap methods can also be used to compare estimation methods and competing models (Arbuckle, 1997). Beyond CFA, these methods can be used with any analysis (e.g., t-test, ANOVA, Correlations, Canonical Correlation Analysis, etc.) and with any statistic (e.g., Roy's largest root, Wilk's lambda, trimmed means).

There are limits to the useful application of bootstrapping methods, however. Bootstrapping should be done only on relatively large samples. As a result, Thompson (1994a) recommended using samples of 40 or larger. Another important limitation is that bootstrapping cannot surpass or correct the limits of the data or the design of a study. Similarly, if you happened to pull a sample of people with an R^2 of .99, you will never get a large standard error, even if the actual population parameter is quite different. Lunneborg (1999) cautions researchers to incorporate knowledge of the design into the interpretations of bootstrap estimates. Bootstrapping cannot turn a descriptive design into an inferential one. Conclusions drawn from bootstrapped results should match the design of the study.

Bootstrap methods are computationally complex, yet the desktop computer used to perform this study computed 2000 bootstrap samples in less that three minutes. More and more computer software is becoming available, as well. Amos can work in conjunction with SPSS. Thompson (1994a) and Rodgers (1999) mention several other programs that bootstrap as well, including EQS, S Plus, Resampling Stats, and SAS (by downloading macro programs from their website).

As Thompson (1996) insisted, "If science is the business of discovering replicable effects, because statistical significance tests do no evaluate result replicability, then researchers should use and report some strategies that do evaluate the replicability of their results" (p. 29). There seems to be little in the way of empirically exploring the internal replicability of obtained results. Increasing awareness that statistical significance tests do not address

replicability will necessitate researchers finding other ways to attend to this important issue.

References

Arbuckle, J. L. (1997). Amos users' guide version 3.6. Chicago: SmallWaters Corp.

Cohen, J. (1994). The earth is round ($p<.05$). American Psychologist, 49, 997-1003.

Diaconis, P., & Efron, B. (1983). Computer-intensive methods in statistics. Scientific American, 248(5), 116-130.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. The Annals of Statistics, 7, 1-26.

Holzinger, K. J., & Swineford, F. (1939). A study in factor analysis: The stability of a bi-polar solution (No. 48). Chicago: University of Chicago.

Lunneborg, C. E. (2000). Data analysis by resampling: Concepts and applications. Pacific Grove, CA: Duxbury.

Rodgers, J. L. (1999). The bootstrap, the jackknife, and the randomization test: A sampling taxonomy. Multivariate Behavioral Research, 34, 441-456.

Thompson, B. (1994a, April). Common methodology mistakes in dissertations, revisited. Paper presented at the annual meeting of the American Educational Research Association, New Orleans. (ERIC Document Reproduction Service No. ED 368 771)

Thompson, B. (1994b). The pivotal role of replication in psychological research: Empirically evaluating the replicability of sample results. Journal of Personality, 62, 157-176.

Thompson, B. (1995). Exploring the replicability of a study's results: Bootstrap statistics for the multivariate case. Educational and Psychological Measurement, 55, 84-94.

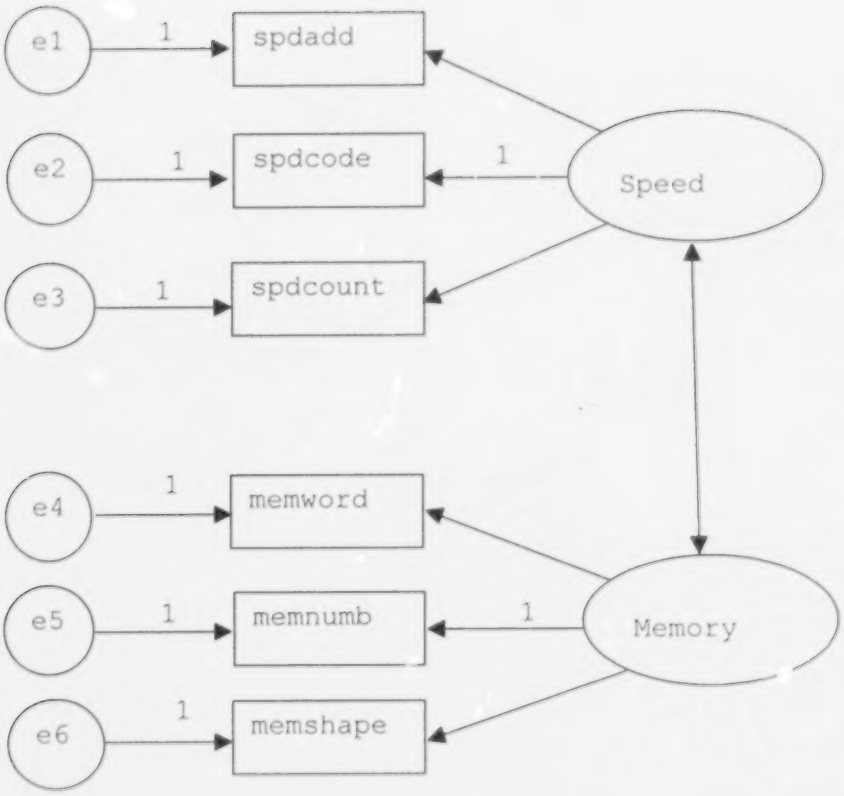
Thompson, B. (1996). AERA editorial policies regarding statistical significance testing: Three suggested reforms. Educational Researcher, 25, 26-30.

Vacha-Haase, T., Kogan, L. R., & Thompson, B. (2000). Sample compositions and variabilities in published studies versus those in test manuals: Validity of score reliability inductions. Educational and Psychological Measurement, 60, 509-522.

Wilkinson, L., & APA Task Force on Statistical Inference. (1999). Statistical methods in psychology journals: Guidelines and explanations. American Psychologist, 54, 594-604. [reprint available through the APA Home Page:

http://www.apa.org/journals/amp/amp548594.html]

Figure1. Input model for confirmatory factor analysis, including six measured variables: Speeded Addition Test (spdadd), Speeded Code Test (spdcode), Speeded Counting of Dots in Shape (spdcount), Memory of Target Words (memword), Memory of Target Numbers (memnumb), and Memory of Target Shapes (memshape).



BEST COPY AVAILABLE

Table 1

Summary of Bootstrapped "Population" Parameters

Factor and Variables	Parameter Estimates			
	B	SE	R^2	SE
Speed				
spdadd	.715	.05	.514	.08
spdcode	.647	.07	.423	.09
spdcount	.644	.05	.417	.09
Memory				
memword	.652	.06	.428	.08
memnumb	.581	.06	.341	.07
memshape	.608	.07	.374	.09

Table 2

Descriptive Statistics of "Population" and Sample Distributions

	Measured Variables					
Statistics	spdadd	spdcode	spdcount	memword	memnumb	memshape
Population (n=301)						
Mean	96.30	69.20	110.50	175.20	90.00	102.50
SD	25.06	15.67	20.25	11.51	7.73	7.63
Skewness	.25	.26	.53	.85	-.09	-.95
Kurtosis	-.27	.49	1.24	1.91	-.17	3.55
Sample (n=75)						
Mean	95.40	69.24	110.47	175.63	89.08	101.65
SD	25.67	14.99	20.71	9.68	7.74	8.23
Skewness	-.09	.20	1.14	-.25	.01	-2.11
Kurtosis	-1.07	.71	3.66	.38	-.19	9.78

Note. The skewness and kurtosis are used in inferential applications of the bootstrap results. The skewness and kurtosis are used in determining the value of the alpha percentage.

Table 3

Summary of Original and Bootstrapped Estimates of Standardized Regression Weights (Pattern Coefficients) for Sample of 75

		Bootstrapped Estimates by No. of Resamples					
		10		200		2000[d]	
Paths	B[a]	B[b]	SE[c]	B[b]	SE[c]	B[b]	SE[c]
spdadd←Speed	.776	.781	.24	.834	.34	.839	.29
spdcode←Speed	.649	.710	.28	.643	.18	.644	.18
spdcount←Speed	.593	.526	.24	.587	.15	.577	.15
memword←Memory	.551	.507	.19	.554	.21	.544	.22
memnumb←Memory	.626	.815	.54	1.808	11.19	3.541	20.03
memshape←Memory	.383	.403	.17	.386	.165	.375	.18

[a]Original estimates without bootstrapping. Amos does not provide standard error estimates. [b]Reflect the mean value of sampling distribution for the statistic. [c]The standard error is the standard deviation of the sampling distribution. It reflects the stability of the estimate over resamples. [d]If using bootstrap for inferential purposes, large numbers of resamples are necessary.

Table 4

Summary of Original and Bootstrapped Estimates of Squared Multiple Correlations (Communality Coefficients) for Sample of 75

Factors and Variables	R^2	Bootstrapped Estimates by No. of Resamples					
		10		200		2000	
		R^2	SE	R^2	SE	R^2	SE
Speed							
Spdadd	.602	.662	.34	.815	1.41	.788	.92
Spdcode	.421	.576	.57	.446	.28	.447	.23
Spdcount	.351	.328	.23	.368	.20	.355	.20
Memory							
Memword	.304	.289	.18	.352	.34	.357	.37
memnumb	.392	.929	1.48	[a]	[b]	[c]	[d]
memshape	.147	.188	.12	.176	.13	.172	.15

[a] R^2=127.80 [b] SE=1297.93 [c] R^2=413.68 [d] SE=3310.83

Table 5

Summary of Original and Bootstrapped Estimates of Error Variance Estimates for Sample of 75

	Original Estimates			Bootstrapped Estimates			
				200 Resamples		2000 Resamples	
Factors and Variables	Total Variance	Error Variance	SE	Error Variance	SE	Error Variance	SE
Speed							
spdadd	658.94	258.52	104.74	115.54	902.17	134.68	587.96
spdcode	224.67	128.24	31.18	120.16	47.09	119.31	46.72
spdcount	428.93	274.47	58.23	261.66	80.33	267.47	74.54
Memory							
memword	93.67	64.37	18.60	58.64	40.25	58.23	32.08
memnumb	59.89	35.91	13.88	[a]	[b]	[c]	[d]
memshape	67.77	57.07	10.95	53.37	19.13	55.58	21.18

[a]Error Variance=-7198.24. [b]SE=74091.50. [c]Error Variance=-2.54e+004.

[d]SE=2.06e+005.